COMP9020 Assignment 1

1.

1. (R1 ; R2) ; R3 = { (a,b) |∃c, (a,c)wpsoffice(R1;R2) ; (c,b)wpsofficeR3}

={ (a,b) |∃c,∃d, (a,d)wpsofficeR1 ; (d,c)wpsofficeR2 ; (c,b)wpsofficeR3}

={ (a,b) |∃d, (a,d)wpsofficeR1 ; (d,b)wpsoffice(R2;R3)}

= R1 ; (R2 ; R3)

(b) R1 = { (a,b) | (a,b)wpsofficeS X S }

= { (a,b) | (a,b) ; (b,b) } = R1 ; I ( Because I = { (x,x) | xwpsofficeS} so (b,b) wpsoffice I )

= { (a,b) | (a,a) ; (a,b) } = I ; R1 ( so on so forth, (a,a) wpsoffice I )

(c) If we assign R1 = { (1,2) } ,R2 = { (2,3) } , then (R1 ; R2)**←** = { (3,1) }, and R1**← ;** R2**←** = wpsoffice, so the statement is not true.

(d) (R1 wpsoffice R2) ; R3 = { (a,b) | ∃c, (a,c)wpsoffice(R1wpsofficeR2) ; (c,b)wpsofficeR3}

= { (a,b) | ( (a1,c1)wpsofficeR1wpsoffice(a2,c2)wpsofficeR2) ) ; ( (c1,b1)wpsofficeR3wpsoffice(c2,b2)wpsofficeR3) ) },

where {(a1,c1)}wpsoffice{(a2,c2)} = {(a,c)} and {(c1,b1)}wpsoffice{(c2,b2)} = {(c,b)}.

= { (a,b)| ( (a1,c1)wpsofficeR1 ; (c1,b1)wpsofficeR3 )wpsoffice( (a2,c2)wpsofficeR2 ; (c2,b2)wpsofficeR3 )}

= (R1 ; R3)wpsoffice(R2 ; R3)

(e) If wen assign R1 = { (1,1) , (1,2) } , R2 = { (1,4) , (1,5) } , R3 = { (1,4) , (2,5) } , then

R1 ; (R2wpsofficeR3) = { (1,4) } , and (R1 ; R2) wpsoffice (R1 ; R3) = { (1,4) , (1,5) } , In this case the left is not equal to the right ,the statement is not true.

2.

1. Base case: Ri = Ri+1 ,which holds Rj = Ri for j = i+1.

Inductive case: Assume Ri+n = Ri+n+1,where nwpsofficeN.So it is obvious Ri+n = Ri+n+1 = Ri+n∪(R;Ri+n) .And Ri+n+2 = Ri+n+1∪(R;Ri+n+1) = Ri+n∪(R;Ri+n) = Ri+n+1 .

For each Rj (j>i ),it is equal to the previous one,thus Rj = Ri for all j≥i.

(b)From the formula:Ri+1 := Ri∪(R;Ri ) i≥0 , we can conclude for every i , Ri wpsoffice Ri+1.So for kwpsoffice[0,i],RkwpsofficeRi.And from question (a),we can get if Ri =Ri+1 , then Ri = Rj for all j≥i ,so RjwpsofficeRi.

In conclusion,Rkwpsoffice Ri for all k≥0.

(c)Base case: R0 ; Rm = Rm (from 1b) = R0+m ,so P(0) holds.

Inductive case: Assume P(n) holds ,which is Rn;Rm = Rn+m.

Rn+1 ; Rm = [Rn∪(R ; Rn)] ; Rm = (Rn ; Rm)∪(R ; Rn ; Rm) (from 1d)

Rn+m+1 = Rn+m∪(R ; Rn+m) = (Rn ; Rm)∪(R ; Rn ; Rm) (from 1d)

So Rn+1 = Rn+m+1,P(n+1) holds.

Therefore P(n) holds for all nwpsofficeN.

(d)We assume (a,b)wpsofficeRk+1 and (a,b)∉Rk.Because Rk+1 := Rk∪(R;Rk ), (a,b)wpsoffice(R;Rk ).

Therefore,∃(a,ck)wpsofficeR, (ck,b)wpsofficeRk.And we can also get (ck,b)∉Ri,iwpsofficek-1(if (ck,b)wpsofficeRk-1,and (a,ck)wpsofficeR,we can get (a,b)wpsofficeRk).So (ck,b)wpsofficeR;Rk-1.

Therefore,∃(ck,ck-1)wpsofficeR ,(ck-1,b)wpsofficeRk-1 , and still (ck-1,b)∉Ri,iwpsofficek-2.

So we get the following result:

(a,ck)wpsofficeR (ck,b) ∉Ri,iwpsofficek-1

(ck,ck-1)wpsofficeR (ck-1,b)∉Ri,iwpsofficek-2

...... ......

(c2,c1)wpsofficeR (c1,b) ∉R0

And c1 ≠ c2 ≠ ...... ≠ck ,ciwpsofficeS, |{c1,c2,......,ck}| = k = |S|.So awpsoffice{c1,c2,......,ck}

(a,b)wpsofficeRk,which conflict with hypothesis(assume (a,b)∉Rk).Therefore,(a,b)wpsofficeRk,which means for every element in Rk+1,it is also in Rk,so Rk+1 = Rk.

(e)From the conclusion of (c),we can assign n = m = k,which is Rk;Rk=R2k.From the conclusion (a) and (d), we can know Rk = Rk+1,and then Rk = Rj for j≥k,so R2k = Rk.Therefore Rk;Rk=Rk,which indicates if (a,b)wpsofficeRk,(b,c)wpsofficeRk,then (a,c)wpsofficeRk.So Rk is transitive.

(f) F = (R∪R<-)

R: Because F0= {(x,x)|xwpsofficeS} and F0wpsofficeFi,so R holds.

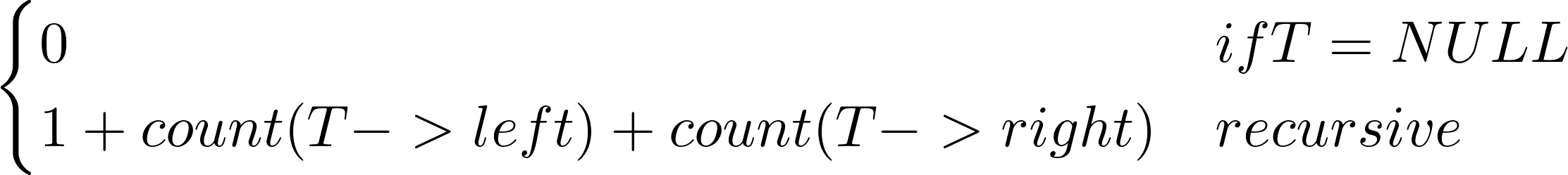
S:I can’t really prove it,but I feel it is certainly right,because R and R<- are symmetric,each time they evolve ,they are still symmetric.

T:It is just the same as the problem (e).

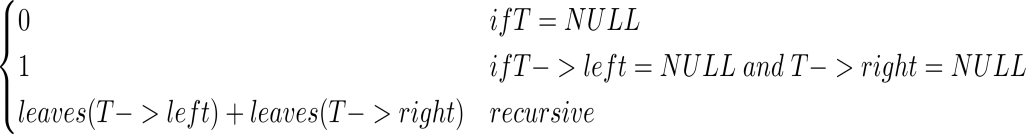
So (R∪R<-) is an equivalence relation.

3.

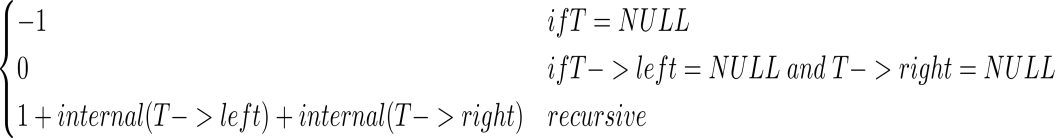
1. a binary tree is either an empty tree represented by a null pointer , or is a single ordered node which contains a data , a left and right pointer and each pointer points to a binary tree.



(b) count(T) =



(c) leaves(T) =



(d) internal(T) =

(e) **Base case:**If the binary tree is only an empty tree which represents NULL,then from the conclusion c and d,internal(T) = -1 and leaves(T) = 0 ,which holds.And if the binary tree’s left pointer and right pointer each points to NULL,we can also see from the c and d that internal(T) = 0 and leaves(T)=1,which also holds.

**Inductive case:** Assume for a arbitary binary tree,internal(T)=I,leaves(T)=L which satisfies L=I+1.Then we consider the following three conditions.

First:We extend a leaf node,making it to be a fully-internal nodes by adding two leaf nodes to be its successors.This time I’ = I+1,L’ = L-1+2=L+1,so L’=I’+1,which holds.

Second:We extend a leaf node,adding one leaf node to be its either left or right successor.This time I’ = I,L’=L-1+1=L,which holds.

Third:We extend a node which is neither a leaf node nor a fully-internal node,making it to be the fully-internal node by adding a leaf node to it.This time I’=I+1,L’=L+1,which holds.

In conclusion,P(T) holds for all binary trees T.

4.

1. h1 = “Alpha uses channel hi” , l1 = “Alpha uses channel lo” ;

h2 = “Bravo uses channel hi” , l2 = “Bravo uses channel lo”;

h3 = “Charlie uses channel hi” , l3 = “Charlie uses channel lo”;

h4 = “Delta uses channel hi” , l4 = “Delta uses channel lo”.

(i) *φ*1 = (h1∨l1)∧(h2∨l2)∧(h3∨l3)∧(h4∨l4)

(ii) *φ*2 =¬(h1∧l1)∧¬(h2∧l2)∧¬(h3∧l3)∧¬(h4∧l4)

(iii)*φ*3 = ((h1∧¬h2)∨(l1∧¬l2))∧((h2∧¬h3)∨(l2∧¬l3))∧((h3∧¬h4)∨(l3∧¬l4))

(b)

(i) If we assign h1 = T,h2=F,h3=T,h4=F,l1=F,l2=T,l3=F,l4=T,this time

V(*φ*1 ∧ *φ*2 ∧ *φ*3) = T.

If we assign h1= T,h2 =T,h3=T,h4=T,l1=F,l2=F,l3=F,l4=F,this time it is quite easy

to know V(*φ*1 ∧ *φ*2 ∧ *φ*3) =F.

So *φ*1 ∧ *φ*2 ∧ *φ*3 is satisfiable for V(*φ*1 ∧ *φ*2 ∧ *φ*3) = T for some truth assignment v.

(ii) First solution : Alpha uses channel hi,Bravo uses channel lo,Charlie uses channel hi,Delta uses channel lo.

Second solution : Alpha uses channel lo,Bravo uses channel hi,Charlie uses channel lo,Delta uses channel hi.